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The basic fuzzy propositional calculus, three important t-norms, fuzzy predicate calculus and a glance for managerial fuzzy implications

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ABSTRACT: This paper aims to study the notion of fuzzy and policy and importance of fuzzy in today's chaotic environment.

***Keywords*:** Fuzzy, policy, Implications, fuzzy policy makers, management

INTRODUCTION

In broad sense, the term "fuzzy logic" has been used as synonymous with "fuzzy set theory and its applications" (Hájek, 1998). Needless to say, the author of the notion of a fuzzy set to which each element belongs in a degree, the characteristic function of such a fuzzy set takes value in the real unit interval [0, 1]. In the narrow sense, fuzzy logic is understood as a kind of mathematical many-valued logic with a comparative notion of truth: sentences can be more or less true. One can study classical logical questions on completeness, decidability, complexity etc. Of the symbolic calculi in question and also possible applications the literature on this *mathematical fuzzy logic* is rather numerous.

Between selecting and not selecting policy, there are other selections. The history of Fuzzy logic and fuzzy method do not go very beyond. But the implementation of fuzzy and its logic in different disciplines are getting increased.

In this area of rapid changing the style of thinking has been changed. Traditionally the thinking method was one dimensional but it has been transferred from one to two, three, four, five and sometimes too many and has been structured a multi facial thinking and method. And this has been the cause of creating creative insights to cope with the facts.

Evidence suggesting that policy and policy making is one of the fundamental parts of any government. And there are different types of policy making process in different countries, for example some countries having democratic and some have dictatorship policy making process (Danaee Fard and Noruzi, 2011).

A glance to fuzzy approach a generalist view

Here we first shall survey the basic fuzzy propositional calculus (Turunen, 1999) The real unit interval [0, 1] is taken to be its standard set of truth values, 1 meaning absolute truth, 0 absolute falsities. The usual ordering ≤ of real's serves as comparison of truth values; we build the logic as a logic with a comparative notion of truth. Continuous t-norms are taken as possible truth functions of conjunction. A binary operation $*$ on [0, 1] is a t-norm if it is commutative ($x * y = y * x$), associative ($x * (y * z) = (x * y) * z$), non-decreasing in each argument (if $x \le x0$ then $x * y \le x0 * y$ and dually) and 1 is a unit element ($1 * x = x$). The t-norm $*$ is a continuous t-norm if it is continuous as a real function. Three most important continuous t-norms are:

$x * y = \max(0, x + y - 1)$ (LÃukasiewicz *t*-norm),
$x * y = \min(x, y)$ (G¨odel *t*-norm),
$x * y = x \cdot y$ (product *t*-norm).

Each continuous t-norm is built from these three in a certain way. The truth function of *implication is* the *residuum of* the corresponding t-norm. If $*$ is your continuous t-norm then its residuum is the operation ⇒ defined as follows:
$x \Rightarrow y = \max\{z | x * z \leq y\}$.
Observe that $x \Rightarrow y = 1$ if $x \leq y$; for $x > y$ the residua of the above *t*-norms are
$x \Rightarrow y = 1 - x + y$ (LÃukasiewicz),
$x \Rightarrow y = y$ (G¨odel),
$x \Rightarrow y = y/x$ (product).

The resulting logic is called BL – the basic fuzzy propositional logic. We sketch its main properties.Work with propositional variables p1, *p*2 . . . and connectives &, → (strong conjunction, implication) and truth constant $\bar{0}$ (falsity). Formulas are defined in obvious way; $\neg\phi$ stands for $\phi \to 0$. Given a continuous t-norm $*$ (and thus its residuum ⇒), each evaluation *e* of propositional variables by truth degrees from [0, 1] extends to an evaluation $e*$ of all formulas; thus
$e*(0) = 0$, $e*(\phi\&\psi) = e*(\phi) * e*(\psi)$, $e*(\phi \to \psi) = e*(\phi) \Rightarrow e*(\psi)$.
Call ϕ a $*$*-tautology if* $e*(\phi) = 1$ for each evaluation *e*; call ϕ a *t- tautology* if it is a $*$-tautology for each $*$ (i.e. However you interpret your propositional variables and connectives, ϕ is true.
The following t-tautologies are taken to be *axioms* of BL:
(A1) $(\phi \to \psi) \to ((\psi \to \chi) \to (\phi \to \chi))$
(A2) $(\phi\&\psi) \to \phi$
(A3) $(\phi\&\psi) \to (\psi\&\phi)$
(A4) $(\phi\& (\phi \to \psi)) \to (\psi\& (\psi \to \phi))$
(A5a) $(\phi \to (\psi \to \chi)) \to ((\phi\&\psi) \to \chi)$
(A5b) $((\phi\&\psi) \to \chi) \to (\phi \to (\psi \to \chi))$
(A6) $((\phi \to \psi) \to \chi) \to (((\psi \to \phi) \to \chi) \to \chi)$
(A7) $\bar{0} \to \phi$
Deduction rule is *modus ponens (*from ϕ and $\phi \to \psi$ infer ψ), *proofs* and provability defined in obvious way.

Completeness: For each formula ϕ, BL proves ϕ if ϕ is a t-tautology. Note completeness for BL, relating provability in BL to tautologicity with respect to algebras called *BL-algebras.* Each continuous t-norm defines a BL-algebra but not conversely.)

In BL we may define derived connectives: $\phi \equiv \psi$ is $(\phi \to \psi) \& (\psi \to \phi)$; min-conjunction $(\phi \wedge \psi)$ is $\phi\& (\phi \to \psi)$, the truth function is minimum, max-disjunction $(\phi \vee \psi)$ is $(((\phi \to \psi) \to \psi) \wedge ((\psi \to \phi) \to \phi)$ (maximum)
And negation $\neg\phi$ *is* $(\phi \to 0)$. The truth function $(-)$ of negation in LÃ is $(-)x = 1 - x$; but the negation of *G* is G¨odel negation: $(-)0 = 1$, $(-)x = 1$ for $x > 0$. Also Π has G¨odel negation.

The three important t-norms defined above (LÃ– LÃ ukasiewicz, G – G¨odel,– product) give us three important and well-known logics stronger than BL:
LÃ ukasiewicz logic can be axiomatized by adding the schema of double nega tion $\phi \equiv \neg\neg\phi$ to BL. Formulas provable in this logic (devoted also by LÃ) are exactly all LÃ -tautologies.
G¨odel logic G is BL plus the schema $\phi \equiv (\phi\&\phi)$ of idemportence of conjunction. Formulas provable in *G* are exactly all *G*-tautologies.
Product logic Π is BL plus two additional axioms $(\phi \to \neg\phi) \to \neg\phi$ *and*
$\neg\neg\chi \to (((\phi\&\chi) \to (\psi\&\chi)) \to (\phi \to \chi))$. (The latter axiom expresses Cancelation by a non-zero element.) Π proves (GEORGE & Yuan, 2008)exactly all Π-tautologies.

Note the importance of two conjunctions & and $\wedge$; they are equivalent only in G¨odel logic, not in other logics. The min-conjunction is idempotent: $\phi \equiv (\phi\wedge\phi)$
is a BL-tautology, the (strong) conjunction & is not, e.g. In LÃ ukasiewicz logic $0.7 * 0.7 = 0.4$, $0.5 * 0.5 = 0$. The formula $(\phi \& (\phi \to \psi)) \to \psi$ is a BL-tautology, but $(\phi \wedge (\phi \to \psi)) \to \psi$
is not (compute its value again in LÃ ukasiewicz for ϕ having the value 0.5 and ψ having the value 0).*

Now we turn to *fuzzy predicate calculus.* Take some predicates *P*1. . . each having its arity (unary, binary,. . .), object variables *x, y, . . .* , connectives &, →, truth constant $\bar{0}$, quantifiers ∀, ∃. (We disregard object constants and function symbols for simplicity.) Formulas are defined in the usual way. An *interpretation* (of *P*1 , . . . , *Pn*) is a structure M = (*M,* (*rP*)*P* predicate) where *M* is a non-empty set (domain) and for each predicate *P* of arity *n, rP* is an n-ary fuzzy relation on *M,* i.e. a mapping associating with each *n*-tuple (*a*1 , . . . , *an*) of elements of M a truth degree rP (a1 , . . . , an) ∈ [0, 1]. The truth value of a formula ϕ in M depends (besides M) on a substitution of free object variables of *ϕ* by elements of *M* and on the chosen semantics of connectives, i.e. On the t-norm ∗. If *α*(*x, ...y*) is a formula and *a, ...b* are elements of *M* then we write *kϕ*(*a, ...b*)*k*∗

for the truth value of the formula *ϕ*(*x, ...y*) under ∗ in the interpretation M for objects a, ...b. It is defined inductively as follows:

$$\|P(a_1 \ldots\ldots a_n)\|^* = rp(a_1 \ldots\ldots a_n)$$
$$\|\varphi \& \Psi\|^*_M = \|\varphi\|^*_M * \|\varphi\|^*_M$$
$$\|\varphi \rightarrow \varphi\|^*_M = \|\varphi\|^*_M \Rightarrow \|\Psi\|^*_M$$
$$\|(\forall x)\varphi(x,b,\ldots)\|^*_M = \inf_{a\in M}\|\varphi(a,b,\ldots)\|^*_M$$
$$\|(\exists x)\varphi(x,b,\ldots)\|^*_M = \sup_{a\in M}\|\varphi(a,b,\ldots)\|^*_M$$

Deduction rules are modus ponens and generalization (from *ϕ* infer (∀*x*)*ϕ*)
– as in classical logic.

BL∀ is complete will respect to general interpretations: BL∀ proves ϕ if *ϕ* is an BL∀-tautology, i.e. for each BL-chain (linearly ordered BL-algebra) L the formula *ϕ* has the L-value 1 for each L-interpretation in which L-values of all formulas are defined. (Such interpretations are called safe and not all interpretations are safe since we cannot assume that the algebra L is suprema and infima of all subsets exist.) Similarly, the predicate versions LÃ ∀, G∀, Π∀ of the corresponding propositional logics are complete w.r.t. Algebras from the corresponding subclasses of the class of BL-chains (called MV-chains, G-chains and product chains fo)r LÃ ukasiewicz, G¨odel and product logic respectively.

With respect to interpretations over [0*,* 1] the situation is more complicated: the set of all predicate t-tautologies (i.e. Formulas being *-tautologies for each continuous t-norm *) is not recursively enumerable (for specialists: it is non- arithmetical). Similarly, neither the set of predicate LÃ ukasiewicz tautologies (tautologies w.r.t.LÃ ukasiewicz t-norm) nor the set of predicate product tautologies is recursively axiomatizable (Samanta and Sarkar, 2005)

Fuzzy decision typology

There is a general view on decision making that it is highly related with information i.e. the amount of information will determine the decision making. The decision table (DT) below consists of a set of mutually exclusive conditions, and refer to a particular actions. Each DT consists of four quadrants:

1. Condition set, if the decision maker or policy maker have the s the condition set consists of all the relevant conditions or attributes (inputs, premises or causes) that have an influence on the decision-making process.
2. Action set, The condition space specifies all possible combinations of condition states of a condition
3. Condition space, the action set contains all the possible actions (outputs, conclusions or consequences) a decision-maker is able to take. This is, the action set points to the possible choice outcome if (for instance) an existing location with a number of specific characteristics is processed through the DT
4. Action space, the action space contains the categorizations of all the possible action states of an action. Any vertical linking of an element from the condition space with an element from the action space produces a decision rule (Wiltox, 2005, p.2) (Figure 1).

Figure 1. The general structure of a decision table

Encountered Problem	
CONDITION SET	CONDITION SPACE
ACTION SET	ACTION SPACE

Suppose we have a set of alternatives such as a set of cities which might be destinations for some future travel plans. Let the set of four decision alternatives be denoted by;

X = {x1, x2, x3, x4} = {Philadelphia, Los Angeles, Chicago, Newark}

Representing our potential destination cities of choice. Note that the set X is a conventional or classical set of objects. We can define a fuzzy subset of the set X, call it A, which is characterized by a membership function μA(xi)

associating with each xi . X a number in the interval [0, 1] which indicates the grade of membership of xi in A. Suppose for our example A is a fuzzy subset defined as:
A = {the city in X is near New York} = {0.6/x 1, 0.001/x2, 0.1/x3, 0.9/x4}
Now the A is fuzzy one (Hagan, 2005, p.2).

In fuzzy we have three different fuzzy decision making conditions as come in the following:

1. Crisp input, crisp process, crisp output like Yager method
2. Fuzzy input, crisp process, crisp output
3. Fuzzy input, Fuzzy process, Fuzzy output like Bonison method (Azar, 2011)

The extension amounts to the introduction of fuzzy sets in the condition and action space of the crisp DT; the crisp condition and action states are replaced with fuzzy conditions and actions. (Azar and Noruzi, 2011)

The importance of Fuzzy thinking in the organizational efficacy

Fuzzy thinking will help the community much more improvement in different aspects for example some come in the following:

- Quality of decision will be increased
- Thinking and the policy will be analyzed from multi faces
- The promotions will be precise
- The evaluations will be precise, because managers and policy makers in the organizations and those who are engaged in selecting people will be evaluate exactly.
- Organizational attitude will be change because staff will not label the others just bad or good
- Economic decisions will be improved by considering fuzzy logic (Vargas Hernandez and Noruzi, 2011).

RESULTS AND DISCUSSION

In this paper, we have studied policy notion based on fuzzy theory to select the most adequate policy. Unlike other decision methods, the fuzzy can adaptively find a suitable policy for the country or the place where policy wants to be implemented (Fathi et al., 2011).

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